Derivation for Y_R*

Here we derive the equation $Y_R^* = Y_R\left(\frac{\phi_i^n}{K_\phi^n + \phi_i^n}\right)$

In our circuits, lets say, one molecule of Y_R combines with n molecules of AI to form Y_R* complex. This reaction can be written as follows:

$$n[AI] + [Y_R] \rightarrow [Y_R^*]$$

So, Rate of forward reaction: $K_1[AI]^n[Y_R]$

Rate of backward reaction: $K_2[Y_R^*]$

Let AI concentration be A

Y_R concentration be R

Y_R* concentration be C

At equilibrium,

Rate of forward reaction = Rate of backward reaction.

$K_1(A - nC)^n(R - C) = K_2C$

If A is very large, $(A - nC) \approx A$

$K_1A^n(R - C) = K_2C$

$K_1A^n(R) - K_1A^nC = K_2C$

$K_1A^n(R) = C(K_1A^n + K_2)$

$$C = \frac{K_1A^n(R)}{K_1A^n + K_2}$$

$$C = \frac{RA^n}{\left(\frac{K_1}{K_2}\right) + A^n}$$

$$C = \frac{R(A)^n}{K + (A)^n}$$

Here our C is concentration of Y_R*, R is concentration of Y_R and A is internal AI concentration (Φ_i).

Hence we get the equation,

$$Y_R^* = Y_R\left(\frac{\phi_i^n}{K_\phi^n + \phi_i^n}\right)$$