Derivation of Y_R*

Here we derive the equation, $Y_R^* = Y_R\left(\frac{\phi_i^n}{K_\phi^n + \phi_i^n}\right)$

LuxR (Y_R) is a DNA-binding protein that binds weakly to the pR promoter and positively regulates the expression of genes downstream to it. On binding to the autoinducer (AI, φ_i), LuxR forms a complex, LuxR* (Y_R*) that activates transcription from the pR promoter.

Say, one molecule of LuxR combines with *n* molecules of AI to form the transcriptional activator complex, LuxR* i.e.,

LuxR + n AI → LuxR*

The rate of the forward reaction is given by $k_f[\varphi_i]^n[Y_R]$ and the rate of the backward reaction by $k_b\ [Y_{R^*}]$.

The simple one step chemical reaction involving LuxR and AI can be assumed to occur at a much smaller timescale than the transcriptional and translational events in the cell. Hence, this reaction is at quasi equilibrium. In the following derivation, the subscript *free* denotes those AI/LuxR molecules that are not present in the bound form in solution. At equilibrium,

$$k_f \phi_{i,\ free}^{\ n} Y_{R,\ free} = k_b Y_R^*$$

$$k_f (\phi_i - nY_R^*)^n\ (Y_R - Y_R^*) = k_b Y_R^*$$

$$k_f (\phi_i)^n\ (Y_R - Y_R^*) = k_b Y_R^* [\phi_i >> nY_R^*]$$

$$Y_R^* = \frac{k_f \phi_i^n Y_R}{k_f \phi_i^n + k_b}$$

$$Y_R^* = \frac{Y_R \phi_i^n}{\left(\frac{k_b}{k_f}\right) + \phi_i^n}$$

$$Y_R^* = Y_R\left(\frac{\phi_i^n}{K_\phi^n + \phi_i^n}\right)$$